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A combined approach to predict friction coefficients and convective heat transfer characteristics in *A* tube with twisted tape inserts for a wide range of *Re* and *Pr*

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Abstract

Generalized correlations are evolved to predict friction factors and convective heat transfer coefficients with twisted tapes in a tube for a wide range of Reynolds numbers and Prandtl numbers. Satisfactory agreement between the present correlations and the data of others validate the proposed correlations. The observations purports the fact that the physical presence of the tape in the flow might lead to monotonic transition from the laminar to turbulent regime. This phenomenon enabled to have a unique correlation for a wide range of Reynolds numbers and Prandtl numbers. Further, a theoretical approach yielded a combined solution for the range of $200 < Re < 10^5$ and $2 < H/D < 10$. The combined friction coefficient correlation valid for a wide range of Reynolds number is further extended to predict theoretically the convective heat transfer coefficients for the overall range of Reynolds number on the basis of the concept that transition might not be present distinctly separating the flow as laminar and turbulent regimes. The theoretical predictions are compared with earlier correlations revealing good agreement between them.

Keywords: Twisted tapes; Combined generalizations; Eddy diffusivity concept; Van Driest's expression

1. Introduction

Convective heat transfer studies with twisted tape inserts in tubes have been the topic of detailed investigations by many researchers. Insertion of tape in the tube is categorized as a passive augmentation technique facilitating efficient thermal energy transportation from the tube wall to the medium flowing in the tubes. Because of the complicated nature of three-dimensional flow arising due to tape insertion the earlier attempts of various investigators are basically performed by applying dimensionless analysis to the system of relevant π-parameters. Bergles [1] presented an excellent review of the augmentation techniques. The earlier theoretical attempts are due Smithberg and Landis [2]. They estimated theoretically the additive contributions of various factors such as thinning of the boundary layer at the wall due to the swirl generated, effect of centrifugal forces in generating secondary flows across the tube, fin effects, etc., leading to augmentation of thermal transport from the wall to the medium. Their theory is compared with their own experimental results related to friction coefficients. Further refinement of their theory is undertaken by Thorsen and Landis [3] for the situation of swirl flow subjected to large transverse temperature gradients. Very recently Sarma et al. [4] visualized the swirl flow as a convective flow in a tube with modified eddy diffusivity equation of van Driest satisfying

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Nomenclature

A^+	damping constant in van Driest expression
C_p	specific heat at constant pressure . . J·kg^{-1}·K^{-1}
D	inside diameter of the tube . . . . m
f	friction coefficient
H	helical pitch of the twisted tape for 360° twist . . . . m
h	heat transfer coefficient . . . . W·m^{-2}·K^{-1}
K	universal constant in van Driest expression
k	thermal conductivity . . . . W·m^{-1}·K^{-1}
m	flow rate . . . . kg·s^{-1}
Nu	Nusselt number, $= hD/k$
Pr	Prandtl number, $= \mu C_p/k$
R	inside radius of the tube . . . . m
R^+	dimensionless radius
Re	Reynolds number, $= 4m/\Pi D\mu$
T	temperature . . . . K
T^+	dimensionless temperature
u	velocity . . . . m·s^{-1}
u^*	shear velocity . . . . m·s^{-1}
u^+	dimensionless velocity, $= u/u^*$
y	distance . . . . m
y^+	dimensionless distance, $= yu^*/\nu$

Greek symbols

ε_h	thermal eddy diffusivity . . . . m^2·s^{-1}
ε_m	turbulent eddy viscosity . . . . m^2·s^{-1}
μ	absolute viscosity . . . . kg·m^{-1}·s^{-1}
ν	kinematic viscosity . . . . m^2·s^{-1}

Subscripts

B	bulk
C	centre line
L	laminar
T	turbulent
W	wall

enhanced coefficients due to the tape insertions in the tube. This concept is tested separately both for the laminar and turbulent ranges of Reynolds numbers for different working media such as water and Turbinol XT-32. The predictions are compared with several correlations existing in the literature indicating reasonable agreement. The recent contributions of Manglik and Bergles [5,6] dealt with heat transfer and pressure drop correlations for twisted tape inserts in two parts for laminar and turbulent flows with transition. Interestingly the momentum transfer studies indicated monotonic relationship for the friction coefficients without any specific transition zone being observed for a wide range of Reynolds numbers. However, the situation is found to be different for convective heat transfer data. In this context the present article puts forth the data of Kishore both for heat transfer and pressure drop obtained for the media such as water and viscous fluids Turbinol-XT 32. The purpose of this article is to check whether the earlier correlations of Sarma et al. [4,7] can be merged into a single correlation satisfying the whole range of Reynolds and Prandtl numbers for tapes tested by them, i.e., for $2 < H/D < 10$ and $200 < Re < 1.2 \times 10^5$. Besides the concept earlier introduced by them might be tangibly employed to evolve a continuous eddy diffusivity expression satisfying wide ranges of Reynolds numbers and Prandtl numbers for different configurations of the tapes defined in terms of H/D.

2. Correlations

In literature many correlations exist for predicting friction coefficients. It can be seen that no two correlations tally with one another except for the orders of magnitude. Kishore [8] gave a detailed recent review. Further, almost all presented separate equations for laminar and turbulent ranges of Reynolds number. Manglik and Bergles [5,6] noted that for flow with tapes the friction coefficient variation with Reynolds number can be represented by a continuous monotonic function. Possibly it can be attributed to the fact that the physical presence of the tape leading to swirl might inhibit the flow instabilities in the transition zone. Sarma et al. [4,7] gave the following correlations:

For $Re < 3000$

$$f_L = 1.5\left[1 + \frac{D}{H}\right]^{3.37} Re^{-0.565} \tag{1}$$

and for $10^4 < Re < 1.2 \times 10^5$

$$f_T = 0.021 Re^{-0.116}\left[1 + \frac{D}{H}\right]^{4.216} \tag{2}$$

Following the procedure suggested by Churchill [9] the two ranges are linked to obtain a continuous curve for the friction coefficient f as follows:

$$f^n = f_L^n + f_T^n \tag{3}$$

The experimental data of Kishore [8] together with Eq. (3) are shown plotted in Fig. 1. Evidently the procedure suggested by Churchill with $n = 5$ lead to a reasonably good correlation. This observation is in support of the conclusion made by Manglik and Bergles [5,6]. In Fig. 2 a generalized correlation is attempted by including the effect of the tape in the ordinate as $f/(1 + D/H)^{3.378}$ versus the abscissa Re, the Reynolds number. Clearly with an allowable scatter of less

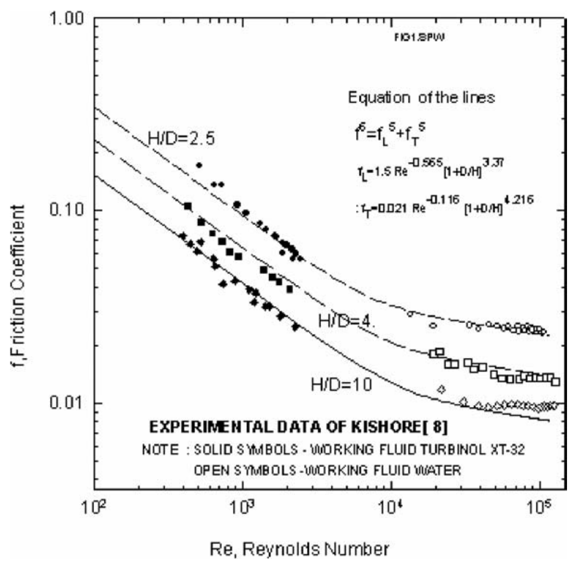

Fig. 1. Variation of friction coefficient with Reynolds number for tape inserts in tubes [$2.5 < H/D < 10$].

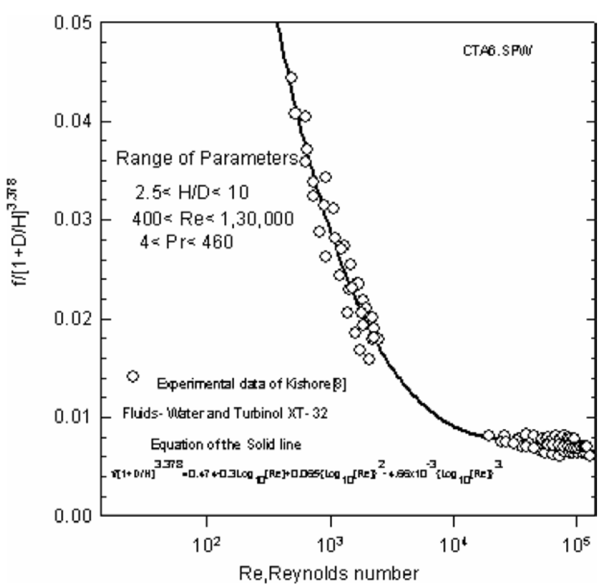

Fig. 2. Generalization of friction coefficient data as function of *Re* and H/D.

than ±12% the data could be correlated by a third degree polynomial. The equation is as follows:

$$\frac{f}{[1+D/H]^{3.378}} = 0.474 - 0.3\log_{10}[Re] + 0.065\{\log_{10}[Re]\}^2 - 4.66\times10^{-3}\{\log_{10}[Re]\}^3 \quad (4)$$

Further the heat transfer data have been plotted comprehensively in a single plot Fig. 3 with the lines derived from the relationship:

$$Nu^5 = Nu_L^5 + Nu_T^5 \quad (5)$$

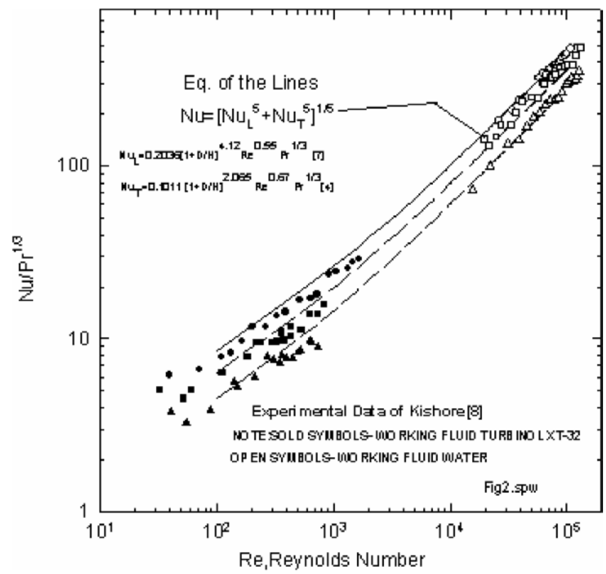

Fig. 3. Heat transfer data of Kishore [8] with the equation lines $Nu = [Nu_L^5 + Nu_r^5]^{1/5}$.

For $100 < Re < 3000$ and $5 < Pr < 400$, Sarma et al. [4, 7] proposed:

$$Nu_L = 0.2036[1+D/H]^{4.12}Re^{0.55}Pr^{1/3} \quad (6)$$

For $10^4 < Re < 1.3\times10^5$; $3 < Pr < 5$

$$Nu_T = 0.1012[1+D/H]^{2.065}Re^{0.67}Pr^{1/3} \quad (7)$$

The details of experimental conditions for data acquisition can be found in [8]. The experimental test section is a tube in a tube heat exchanger with tape inserts located in the inner tube. The heating of the medium is accomplished with hot fluid flowing in the annular passage. Computing the logarithmic mean temperature difference between inlet and outlet, the overall heat transfer coefficient is estimated. The working media are water and viscous fluid Turbinol XT-32. The correlations of Sarma et al. [4,7] are chosen for the combined analysis of the convective heat transfer with tapes. The results are shown in Fig. 4 to have a combined correlation for the whole range of *Re* and *Pr*. The data are correlated by a third degree polynomial with a scatter less than ±12%:

$$\log_{10}\left[\frac{Nu}{Pr^{1/3}(1+D/H)^2}\right] = b_0 + b_1\log_{10}[Re] + b_2\{\log_{10}[Re]\}^2 + b_3\{\log_{10}[Re]\}^3 \quad (8)$$

where $b_0 = 0.974$, $b_1 = -0.783$, $b_2 = 0.35$ and $b_3 = -0.0273$.

To establish the utility of the correlation a search is conducted in the literature and the available data of Lecjaks et al. [10] for different working fluids such as glycerol and glycerol–water mixtures are chosen and plotted in the same coordinate system together with the data points of Kishore [5] in Fig. 5. The solid line is the polynomial equation (8) passing through the data for the range 200 <

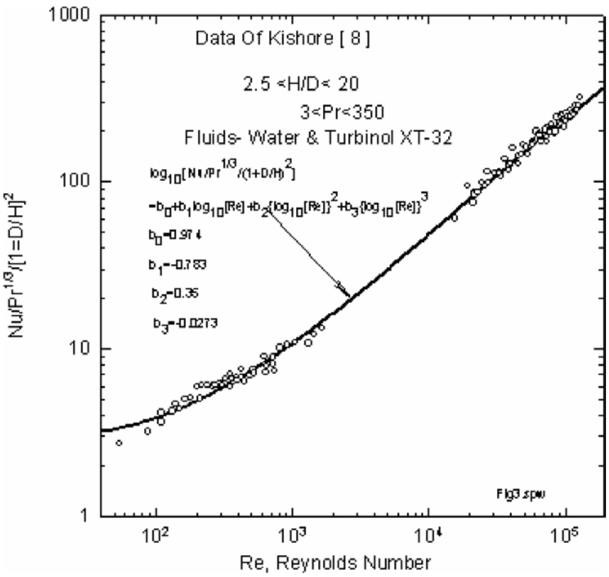

Fig. 4. Generalization of heat transfer data for $2.5 < H/D < 20$.

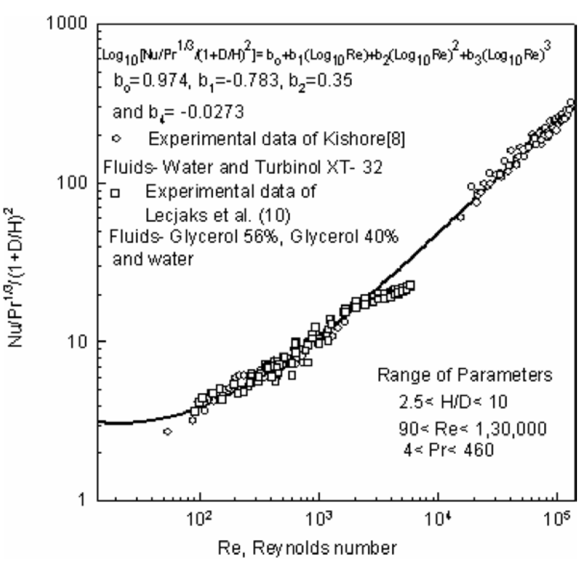

Fig. 5. Comparison of Lecjaks et al. [10] with present correlation.

$Re < 1.3\text{e}5$, $2 < Pr < 400$. The distinct clustering of the data points along the polynomial line fortifies the utility of Eq. (8) for a full range of Re and Pr satisfying one of the aims of the article.

3. Theoretical analysis

Having generalized the data of friction coefficients and the Nusselt numbers for practicable ranges of the system parameters, further attempt is made to check the validity of the earlier concepts proposed by Sarma et al. [4,7]. The twisted tape studies leading to enhancement of convective heat transfer can be analyzed considering that a modification of eddy diffusivity of van Driest holds good. The universal constant K in the eddy diffusivity is to be considered as a function of Re and H/D. The van Driest's eddy diffusivity is given by the expression [11]:

$$\frac{\varepsilon_m}{\nu} = \left\{Ky^+\left[1 - \exp\left(-y^+/A^+\right)\right]\right\}^2 \left|\frac{\partial u^+}{\partial y^+}\right| \tag{9}$$

From the dimensionless generalizations, i.e., Eqs. (4) and (8) it can be inferred that K in Eq. (9) be obtained as continuous functions of Re and H/D. Hence subsequent analysis is devoted to predict theoretically the Nusselt numbers assuming the friction coefficient equation (1) is known a priori. Following the assumptions as employed in previous studies, a numerical procedure is outlined.

The velocity field is fixed from the differential equation:

$$\frac{\partial u^+}{\partial y^+} = \frac{1 - y^+/R^+}{1 + \varepsilon_m/\nu} \tag{10}$$

where ε_m/ν is given by Eq. (9) and

$$R^+ = \frac{Re}{2}\sqrt{\frac{f}{2}} \tag{11}$$

The boundary condition for solving Eq. (10) is $u^+ = 0$ at $y^+ = 0$.

Thus the value of K in Eq. (9) is determined numerically by employing an iterative procedure as follows:

(1) Prescribe the system conditions, i.e., Re, H/D with an assumed value to K of Eq. (9). The value of Re can be from a low magnitude say 200. Calculate R^+ from Eq. (11) with the aid of Eq. (3).
(2) Solve the differential equation (10) to obtain $u^+ = f(y^+)$. Hence estimate Re from the law of continuity in dimensionless form as follows

$$Re = \frac{4m}{\pi D\mu} = 4\int_0^{R^+} u^+\left(1 - y^+/R^+\right) \mathrm{d}y^+ \tag{12}$$

(3) Check the value of Re with the pre-assigned value in step 1. If the values coincide then the eddy diffusivity can be correctly predicted for the given system conditions from Eq. (9). Subsequently, the value is used in the energy equation to predict wall temperature gradient. Otherwise an iterative procedure is employed till $Re_{\text{assigned}} = Re_{\text{calculated}}$ from Eq. (12). Assume the momentum eddy diffusivity is same as thermal energy diffusivity for $Pr > 1$, i.e., $\varepsilon_h = \varepsilon_m$. In all the iterations the damping constant $A^+ = 26$ and iteration is conducted to fix the value of K as a function of Re and H/D. Reynolds number is assumed in steps of 200 to reach a target value of 1.2×10^5. For each step the iterative procedure is performed.

4. Temperature profiles

On the assumption that the profile across the tube is fully developed the temperature profile can be obtained from the dimensionless differential equation

$$\frac{\mathrm{d}}{\mathrm{d}y}\left[\left(1+\frac{\varepsilon_h}{\nu}Pr\right)\frac{\mathrm{d}T^+}{\mathrm{d}y^+}\right]=0 \tag{13}$$

where $T^+=(T_\mathrm{W}-T)/(T_\mathrm{W}-T_\mathrm{C})$ and $y^+=yu^*/\nu$.

The boundary conditions are:

at $y^+=0, \quad T^+=0$
at $y^+=R^+, \quad T^+=1$

Thus, the temperature profile will be utilized to evaluate the heat transfer coefficient:

$$h=-\frac{k}{(T_\mathrm{W}-T_\mathrm{B})}\frac{\mathrm{d}T^+}{\mathrm{d}y^+}\bigg|_{y^+=0} \tag{14}$$

Eq. (14) in dimensionless form is written as:

$$Nu_m=\frac{hD}{k}=2R^+\frac{\mathrm{d}T^+}{\mathrm{d}y^+}\bigg|_{y+=0}\frac{T_\mathrm{W}-T_\mathrm{C}}{T_\mathrm{W}-T_\mathrm{B}} \tag{15}$$

The temperature correction ratio in Eq. (15) can be calculated with aid of temperature and velocity profiles as follows:

$$\frac{T_\mathrm{W}-T_\mathrm{C}}{T_\mathrm{W}-T_\mathrm{B}}=\frac{\int_0^{R^+}u^+(1-y^+/R^+)\,\mathrm{d}y^+}{\int_0^{R^+}u^+T^+(1-y^+/R^+)\,\mathrm{d}y^+} \tag{16}$$

Thus following the procedure outlined the combined solution for a wide range of $Re-200-1.2\times10^5$ is obtained. The results are shown plotted in Figs. (6)–(8) between $Nu/Pr^{1/3}$ versus Re for $Pr=3$ (water). Besides on the same plot the dimensionless correlations of Sarma et al. [4,7] are also shown plotted. The trends from the analytical solution fairly agree with the correlations already derived from experimental data. In Fig. 9 the variation of K in eddy diffusivity equation is shown plotted. Evidently eddy diffusivity is found to be dependent on Re and H/D. The results indicate that as H/D decreases the magnitude of K increases for a given Re. It can be attributed to the fact that increase in swirl promotes higher values of ε_m/ν leading to enhancement of heat transfer rates. Thus the following conclusions can be derived from the study performed.

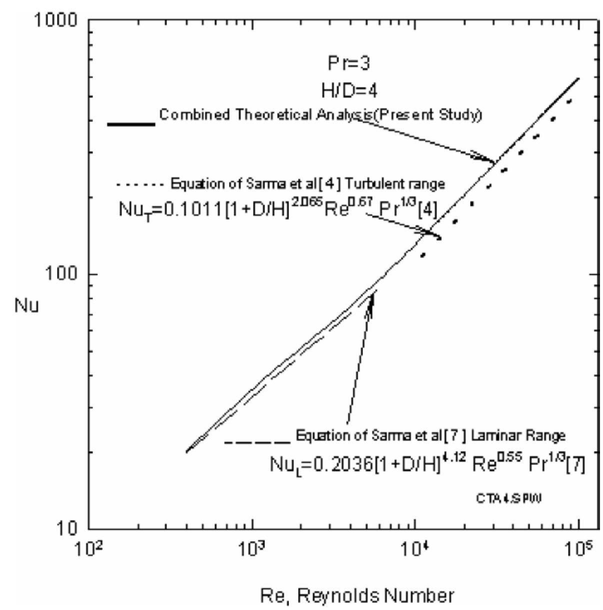

Fig. 7. Comparison of present combined theory with the earlier correlations $H/D=4$.

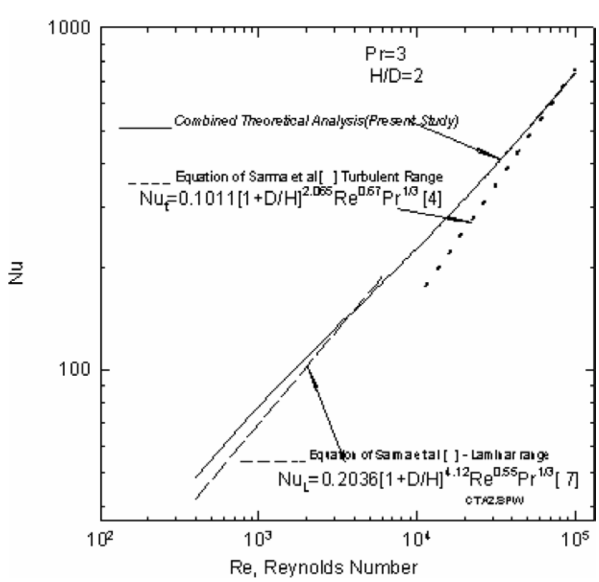

Fig. 6. Comparison of present combined theory with the earlier correlations $H/D=2$.

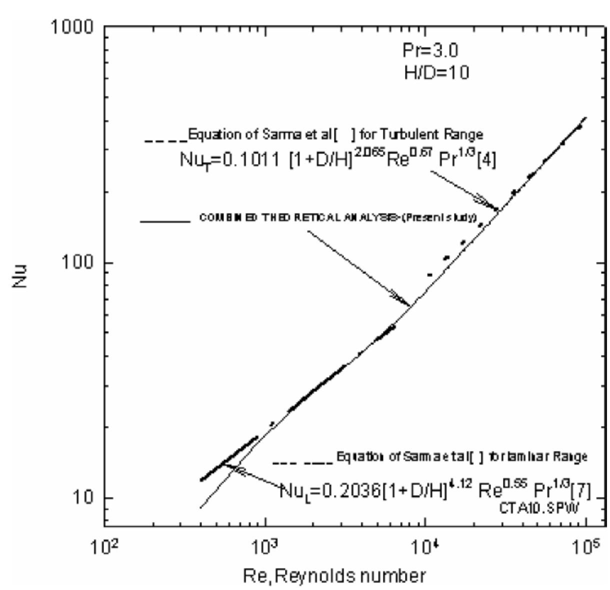

Fig. 8. Comparison of present combined theory with the earlier correlations of $H/D=10$.

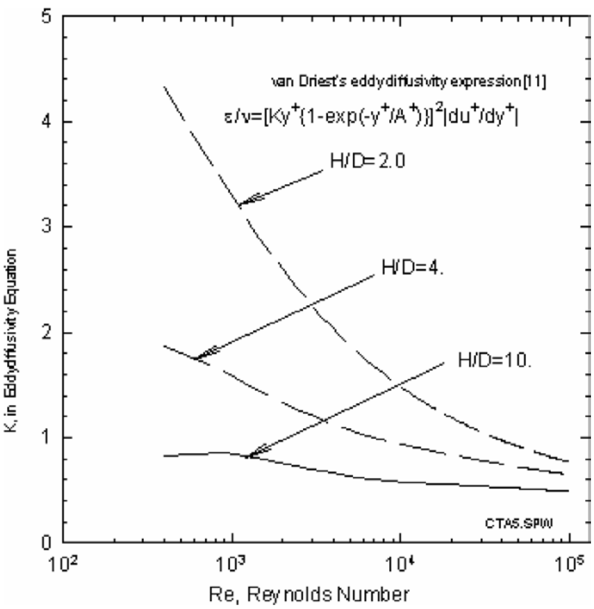

Fig. 9. Variation of Kin Eddy diffusivity equation of van Driest with Reynolds number.

5. Conclusions

(1) The present study could successfully encompass the whole range of *Re* and pitch to diameter ratios under the framework of unified theory both from dimensionless and analytical consideration.
(2) Generalized equations are provided through Eqs. (3) or (4) for the friction coefficients as functions of *Re*. Lumping the complex flow effects on to one of to one of the two variables in the van Driest's expression proved successful. Thus the classical expression of van Driest has a potential to answer both laminar and turbulent ranges of *Re* with a suitable modification to the constant K.
(3) Similarly the convective heat transfer can be predicted with aid of Eqs. (5) or (8). The ranges of applicability are as follows: $200 < Re < 1.2 \times 10^5$; $3 < Pr < 450$; $2 < H/D < 10$.
(4) The theoretical solution could be reasonably derived as a combined monotonic dependence to yield $Nu = F[Re, H/D, Pr]$ by considering $K = F\{Re, H/D]$ in the eddy diffusivity expression of van Driest. This parameter is not only useful to predict convective heat transfer but also for a situation with tape inserts in a tube with the aid of plot, i.e., Fig. 9. From the trends of the results it can be inferred that the relationship f vs. *Re* is monotonic and an abrupt jump in the characteristic is not to be observed. In all likelihood the presence of the tape might inhibit the transitional jump.

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